't Hooft GR Exercise 5.1 pg 21

$x \to u$: $\Gamma^{\nu}_{k\lambda}(x) \to \tilde{\Gamma}^{\nu}_{k\lambda}(u) = u^{\nu}_{,\mu}\left[x^{\alpha}_{,k}\, x^{\beta}_{,\lambda}\, \Gamma^{\mu}_{\alpha\beta}(x) + x^{\mu}_{,k,\lambda}\right]$

$u \to y$: $\tilde{\Gamma}^{\nu}_{k\lambda}(u) \to \tilde{\Gamma}^{\nu}_{k\lambda}(y) = y^{\nu}_{,\mu}\left[u^{\alpha}_{,k}\, u^{\beta}_{,\lambda}\, \Gamma^{\mu}_{\alpha\beta}(u) + u^{\mu}_{,k,\lambda}\right]$

$$\Rightarrow \tilde{\Gamma}^{\nu}_{k\lambda}(y) = y^{\nu}_{,\mu}\left[u^{\alpha}_{,k} u^{\beta}_{,\lambda} u^{\mu}_{,\rho}\left[x^{\sigma}_{,\alpha}\, x^{\varepsilon}_{,\beta}\, \Gamma^{\rho}_{\sigma\varepsilon}(x) + x^{\rho}_{,\alpha,\beta}\right] + u^{\mu}_{,k,\lambda}\right]$$

$$= \frac{\partial y^{\nu}}{\partial u^{\mu}}\left[\frac{\partial u^{\alpha}}{\partial y^{k}}\frac{\partial u^{\beta}}{\partial y^{\lambda}}\frac{\partial u^{\mu}}{\partial x^{\rho}}\left[\frac{\partial x^{\sigma}}{\partial u^{\alpha}}\frac{\partial x^{\varepsilon}}{\partial y^{\beta}}\Gamma^{\rho}_{\varepsilon\sigma}(x) + \frac{\partial^2 x^{\rho}}{\partial u^{\alpha}\,\partial y^{\beta}}\right] + \frac{\partial^2 u^{\mu}}{\partial y^{k}\,\partial y^{\lambda}}\right]$$

$$= \frac{\partial y^{\nu}}{\partial x^{\rho}}\left[\delta^{\sigma}_{k}\,\delta^{\varepsilon}_{\lambda}\cdot\Gamma^{\rho}_{\varepsilon\sigma}(x) + \frac{\partial^2 x^{\rho}}{\partial y^{k}\,\partial y^{\lambda}}\right].$$

$$\boxed{= y^{\nu}_{,\rho}\left[\Gamma^{\rho}_{k\lambda}(x) + x^{\rho}_{,k,\lambda}\right]}$$

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